API - esercitazione 5 30/03/2021

Riepilogo grammatiche

minore potere generat.

Tipo	Nome	Produzioni	Automi
3	Regolari	$A \to aB \mid a \mid \varepsilon$	FSA
2	Context-free	$A \to \beta$	NPDA
1	Context-sensitive	$\alpha \to \beta,\ \lvert\alpha\rvert \leq \lvert\beta\rvert$	Linear Bounded Automata
0	Non ristrette	$\alpha \to \beta$	TM

maggior potere generativo

$A, B \in V_n$, $a \in V_t$, $\beta \in V^*$, $\alpha \in V_n^+$

Riepilogo formalismi:

minor potere espressivo

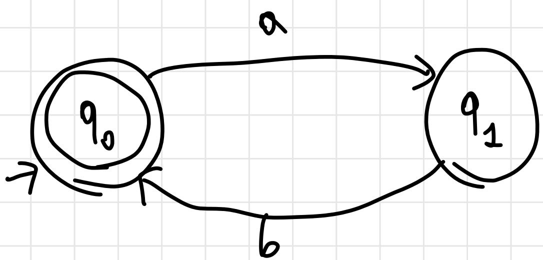
- FSA NFSA Gr. Reg. REGEX
- PDA
- NPDA Gr. C.F.
- TM NTM Gr. non ristrette

maggior potere espressivo

Pattern grammatiche regolari:

non-terminali da associare ad un significato

es.:



$$\Rightarrow \quad S \to a Q_1 \mid \varepsilon$$
$$Q_1 \to b S$$

non-terminali $\equiv$ stati

(nelle gr. reg.)

Pattern gr. CF:

$L = a^n b^n, \ n \geq 0$ $\Rightarrow$ $S \to aSb \mid \varepsilon$

tipico pattern CF per il conteggio

$S \to bSb$

es.:

$L: \quad x \in (a|b)^+ \ \wedge \ \#_a(x) = \#_b(x)$

$S \rightarrow aSb \mid bSa \mid ab \mid ba$

alternativamente:

$S \rightarrow aGb \mid bGa$

$G \rightarrow aGb \mid bGa \mid \varepsilon$

abb|aab

$a^{-1} = b \ , \ b^{-1} = a$

$w(w^R)^{-1}$

abba

Proposta :

$S \to aSb \mid bSa \mid abS \mid baS \mid \varepsilon$

Contraes. :

$aabbba \in L$

$$S \to aGbG \mid bGaG$$

$$G \to aGbG \mid bGaG \mid \varepsilon$$

$$L = x \in (a|b)^+ \wedge \#_a(x) = \#_b(x)$$

$$\underbrace{w(w^R)^{-1}} \cdot u(u^R)^{-1} \cdot x(x^R)^{-1}$$

$$abbbaaabbaab \in L$$

$$a^{-1} = b$$

$$b^{-1} = a$$

$$S \to (S)S \mid \varepsilon$$

Dyck language (forse)

es.:

$$L = \{ ab^{n_1} ab^{n_2} \dots ab^{n_k} \mid \forall i : n_i > 0 \wedge k \geq 2$$
$$\wedge\ \exists j, h \left(1 \leq j < h \leq k \wedge n_j = n_h\right)\}$$

$$abbb\ ab\ abbb\ abb \in L$$

$$L = (ab^{+})^{*}\, ab^{n}\, (ab^{+})^{*}\, ab^{n}\, (ab^{+})^{*}, \quad n \geq 1$$

$$L = \underbrace{(ab^+)^*}_{G} \; \overbrace{ab^n \; (ab^+)^* \; ab^n} \; \underbrace{(ab^+)^*}_{G} \;, \quad n \geq 1$$

$$S \to G\, a X \, G \qquad\qquad X \Rightarrow ab^n \, (ab^+)^* \, ab^n$$

$(ab^+)^*$
$$\begin{cases} G \to aH \mid \varepsilon \\ H \to bH \mid bG \end{cases}$$

$$S \Rightarrow G a X G$$

$$X \to bXb \mid Ga$$

$$\Rightarrow G\, ab X b\, G \Rightarrow G\, abbXbb\, G$$

$$\Rightarrow G\, abbbXbbb\, G \Rightarrow G\, abbbaGbbb\, G$$

c'è ancora un problema

$$\Rightarrow (ab^+)^* \, abbb\,a\,(ab^+)^* \, a\,bbb\,(ab^+)^*$$

$S \to G\, a\, X\, G$

$(ab^+)^*$ $\left[\begin{array}{l} G \to aH \mid \varepsilon \\ H \to bH \mid b\,G \end{array}\right.$

$X \to bXb \mid bGab$

$S \Rightarrow GaXG$

$\Rightarrow GabGabG$

$\Rightarrow GababG$

es.: $L = w\,(0|1|\varepsilon)\,w^R , \quad w \in \{0,1\}^*$

$$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$$

$$S \Rightarrow 0S0 \Rightarrow 01S10 \Rightarrow 011S110 \Rightarrow 0110110$$

una gram. è giusta se genera tutte e sole le stringhe di L

$$\forall x \, (x \in L \Rightarrow \exists\, S \Rightarrow^* x) \qquad (\text{tutte})$$

$$\wedge$$

$$\forall x \, (\exists\, S \Rightarrow^* x \Rightarrow x \in L) \qquad (\text{sole})$$

Dim. $\forall x \, (x \in L \Rightarrow \exists S \Rightarrow^* x)$ $\qquad L = w(0|1|\varepsilon)w^R$

$w \in \{0,1\}^*$

Induzione sulla lunghezza delle stringhe

$\exists ? \Rightarrow^* \varepsilon$

$S \Rightarrow \varepsilon$ ✓

$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$

Passo induttivo:

Hp. ind. $\forall y \, (y \in L \land |y| = k-1 \Rightarrow \exists S \Rightarrow^* y)$

Th. $\forall x \, (x \in L \land |x| = k \Rightarrow \exists S \Rightarrow^* x)$

$|x| = k \land x \in L$

$$x = \begin{cases} ww^R & \text{lunghezza pari} \\ waw^R, \ a \in \{0,1\} & \text{dispari} \end{cases}$$

Hp. ind. $\forall y \left(y \in L \wedge |y| = k-1 \Rightarrow \exists S \Rightarrow^* y \right)$ ✓

Th. $\forall x \left(x \in L \wedge |x| = k \Rightarrow \exists S \Rightarrow^* x \right)$

k dispari : $x = w \, a \, w^R$ $\qquad a \in \{0,1\}$

k-1 pari $y = w \, w^R$

$S \to \underbrace{0S0 \mid 1S1 \mid \underbrace{0 \mid 1 \mid \varepsilon}}$

Per Hp. ind. $\exists S \Rightarrow^* w \, w^R$

$\exists S \Rightarrow^* \underbrace{?} \Rightarrow w w^R$ $\qquad \exists S \Rightarrow^* w \, S \, w^R$

$\exists S \Rightarrow^* \underbrace{w \, S \, w^R} \Rightarrow w \, w^R$ $\qquad \Rightarrow w \, a \, w^R = x$

$(S \to \varepsilon)$

Hp. ind. $\forall y \left(y \in L \land |y| = k-1 \Rightarrow \exists S \Rightarrow^* y \right)$

Th. $\forall x \left(x \in L \land |x| = k \Rightarrow \exists S \Rightarrow^* x \right)$

k pari: $x = w\,w^R = \underbrace{u\,a}_{w}\ \underbrace{a\,u^R}_{w^R}$, $a \in \{0, 1\}$

k-1 dispari: $y = u\,a\,u^R$ $\begin{bmatrix} u\,0\,0\,u^R \\ u\,1\,1\,u^R \end{bmatrix}$

$S \to \overbrace{0S0 \mid 1S1}\mid \underbrace{0 \mid 1 \mid \varepsilon}$

Per Hp. ind.

$\exists S \Rightarrow^* u\,a\,u^R$

$\exists S \Rightarrow^* ? \Rightarrow u\,a\,u^R$

$\exists S \Rightarrow^* u\,S\,u^R \Rightarrow u\,a\,u^R$

$\exists S \Rightarrow^* u\,S\,u^R \Rightarrow$

$\Rightarrow u\,a\,S\,a\,u^R \Rightarrow u\,a\,a\,u^R$

$= w\,w^R = x$

Dim. $\forall x \left(\exists S \Rightarrow^{*} x \Rightarrow x \in L \right)$

$S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$

$\exists S \Rightarrow^{k} x \Rightarrow x \in L$

$\exists S \Rightarrow^{1} x \Rightarrow x \in L$ caso base

$0 \overset{?}{\in} L$ ✓

$1 \overset{?}{\in} L$ ✓

$\varepsilon \overset{?}{\in} L$ ✓

$L = w \, (0 \mid 1 \mid \varepsilon) \, w^R$

Hp. ind. $\forall y \left(\exists S \Rightarrow^{k} y \Rightarrow y \in L \right)$ $\qquad a \in \{0,1\}$

Th. $\forall x \left(\exists S \Rightarrow^{k+1} x \Rightarrow x \in L \right)$

$\exists S \Rightarrow^{k} y \;,\; y \in L \qquad y = w(0|1|\varepsilon)w^R$

$$\exists S \Rightarrow^{k-1} \underbrace{wSw^R}_{} \underset{(k)}{\Rightarrow} w\underbrace{(0|1|\varepsilon)}_{S}w^R$$

$$\exists S \Rightarrow^{k-1} wSw^R \underset{(k)}{\Rightarrow} waSaw^R \underset{(k+1)}{\Rightarrow} \overbrace{wa}^{z}(0|1|\varepsilon)\overbrace{aw^R}^{z^R} \in L \quad \checkmark$$

$S \rightarrow \overbrace{0S0 \mid 1S1}^{} \mid \overbrace{0 \mid 1 \mid \varepsilon}^{}$

es.: $L = \{ a^n b^{2n} c^{n/2} \mid n \geq 1 \}$ (Arrotondata per dif.)

$S \to a\,G\,B\,C\,E$

$G \to a\,G\,B\,C$

$CB \to BC$ (swap rule)

$CE \to F$

$CF \to Ec$

$F \to \varepsilon$

$aaaBCBBEc$

$aaaBBCBEc$

$aaaBBBCEc$

$aaaBBBFc \to aaaBBBc$

$aaaBCBCBC$

$aaaBBBCCC$

$aaaBCBCBCE$

$aaaBCBCBF$

$aaaBCBBCF$

$S \to a\,G\,B\,C\,E$

$G \to a\,G\,B\,C \mid \varepsilon$

$CB \to BC$ (swap rule)

$CE \to F$

$CF \to Ec$

$F \to \varepsilon$

$B \to bb$

$E \to \varepsilon$

$aa\,BC\,BC\,E$

$aa\,BC\,bb\,C\,E$

$aa\,BC\,bb\,F$

$aa\,BC\,bb$

$aa\,bb\,C\,bb$

aa
bbbb
c

sulle slide è diversa

es. da TdE: si scrivano Automa e Gramm.

$$L = \{ a^n a^m a^m a^n \mid n, m \geq 1,\ n \text{ pari},\ m \text{ dispari} \}$$

FSA PDA NPDA TM

NB.: Automa a pot. ric. min.

Gram. minor n° di prod.

$$a^{n+m+m+n} = a^{2n+2m} = a^{2(n+m)}$$

$a^{2(n+m)}$ $\quad n, m \geq 1$, n pari, m dispari

$n = 2$, $m = 1$ $\qquad\qquad$ $n = 2$ $\quad m = 3$

$a^{2(3)} = a^6$ $\qquad\qquad$ $a^{2(5)} = a^{10}$

$n = 4$, $m = 1$ $\qquad\qquad$ $n = 4$, $m = 3$

$a^{2(5)} = a^{10}$ $\qquad\qquad$ $a^{\overset{2(7)}{n+m}} = a^{14}$

$a^{2 \cdot 9}$ $\quad a^{2 \cdot 11}$ $\quad \ldots$

$n = 6 \quad m = 3$

$a^{2(9)} = a^{18}$

$n = 6 \quad m = 5$

$a^{2(11)} = a^{22}$

$L = a^{2(l)} \quad , \quad l \geq 3 \quad , \quad l$ dispari

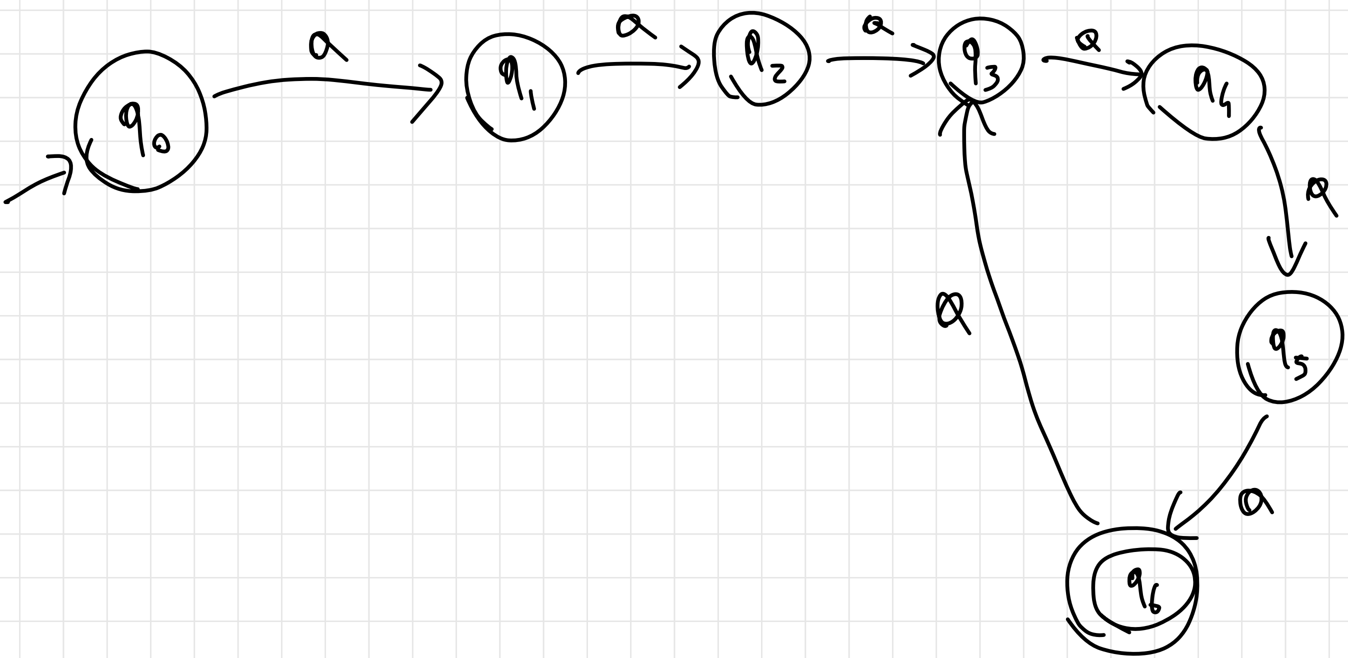
$= a^{2(2k+1)} \quad , \quad k \geq 1$

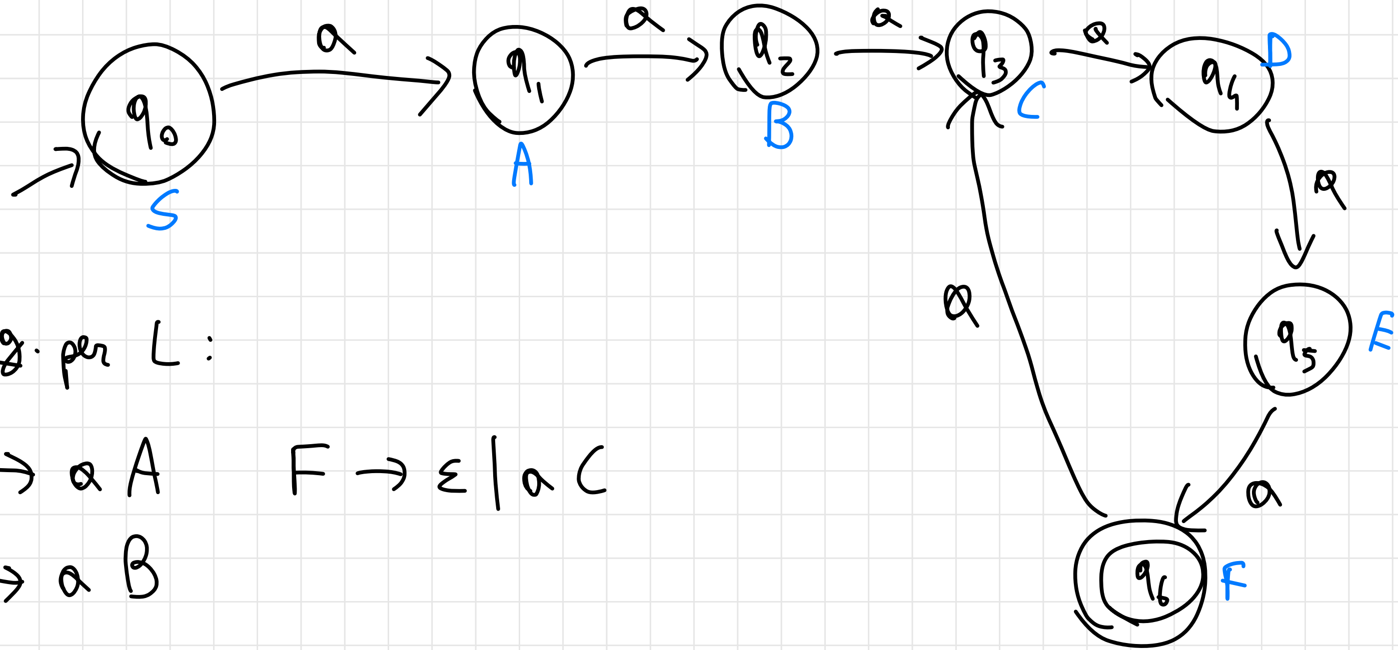
$= a^{4k+2} \quad , \quad k \geq 1$

a^6

$a^6 \quad a^{10} \quad a^{14} \quad \ldots$

a^{4k+2} , $k \geq 1$





Gr. reg. per L :

$S \to aA$

$A \to aB$

$B \to aC$

$C \to aD$

$D \to aE$

$E \to aF$

$F \to \varepsilon \mid aC$