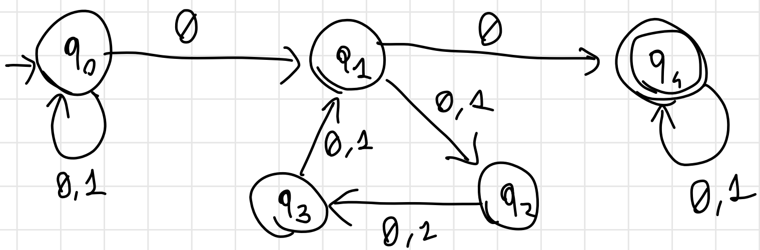
API - esercitazione 4

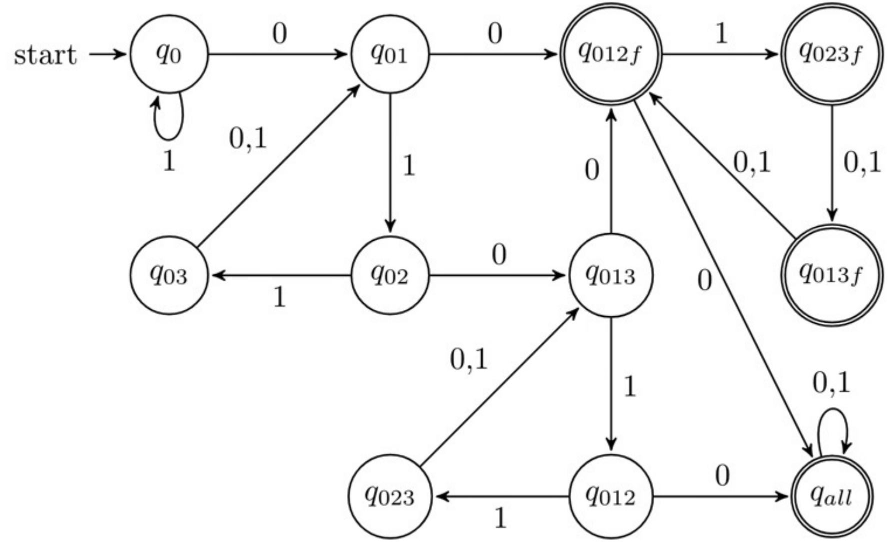
23/03/2021

$$Q \rightarrow P(Q)$$

NFSA:

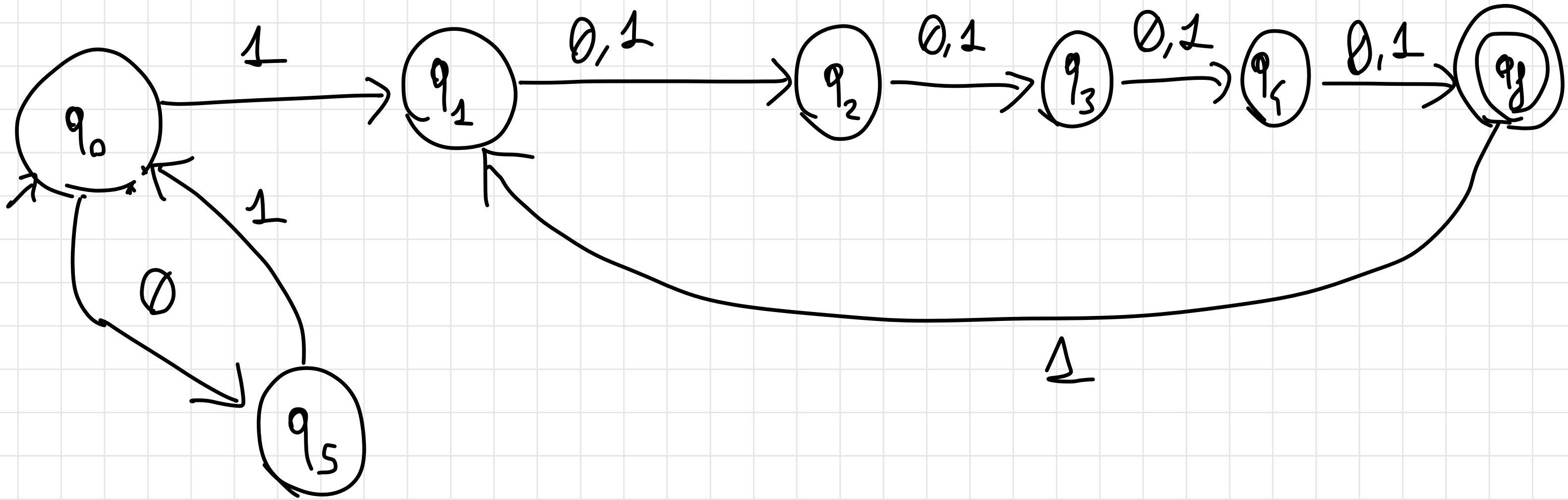


FSA:



es.:

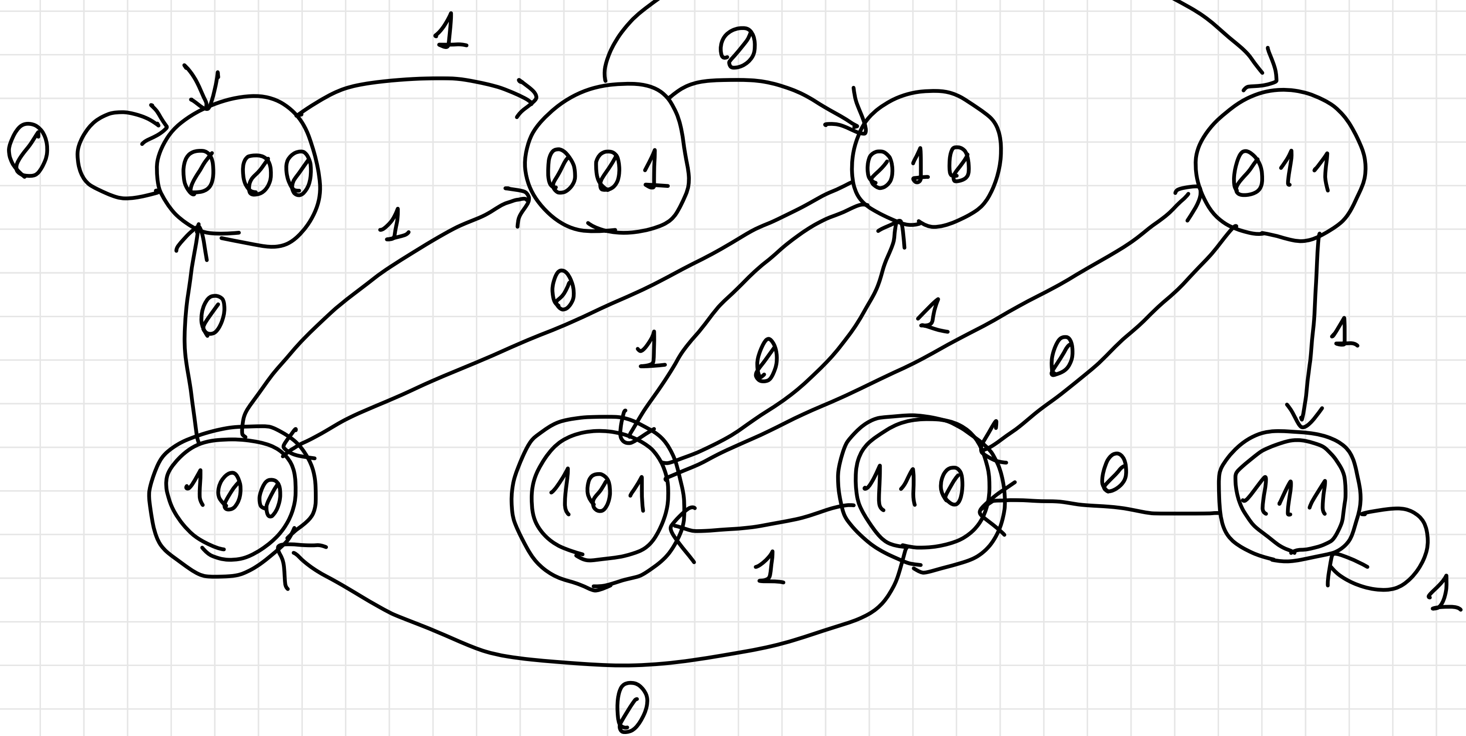
$$L = (0|1)^* 1 (0|1)^4$$

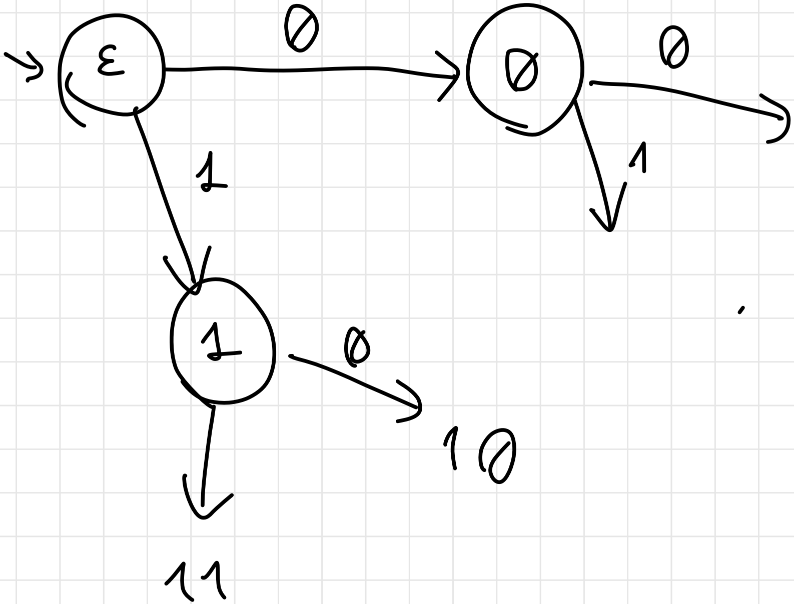


00000 00001 00010 ...

$$2^5 = 32$$

$L_1 = (0|1)^* 1 (0|1)^2$





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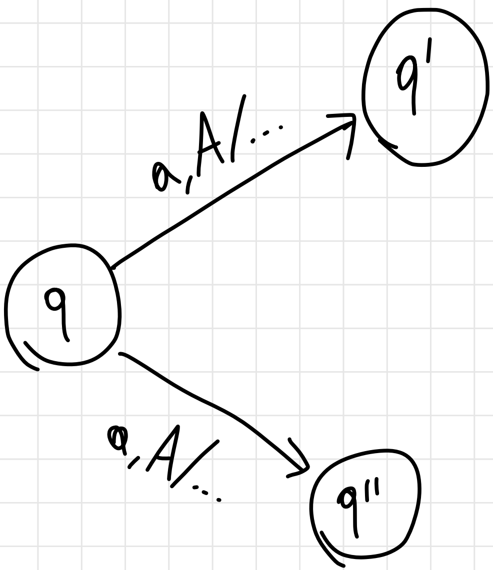
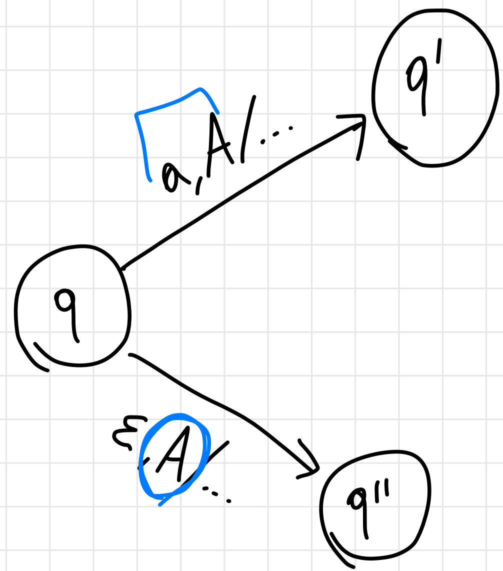
001

NPDA

Fonti di non det.:

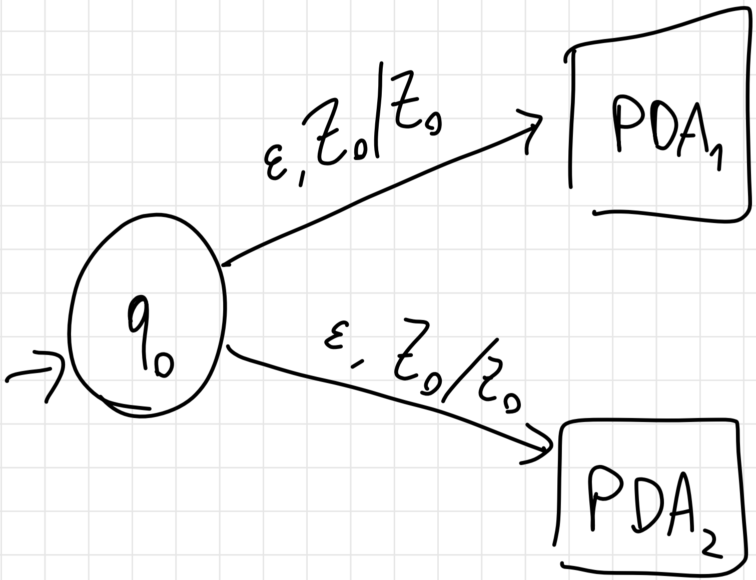




NPDA espressivamente più potenti dei PDA

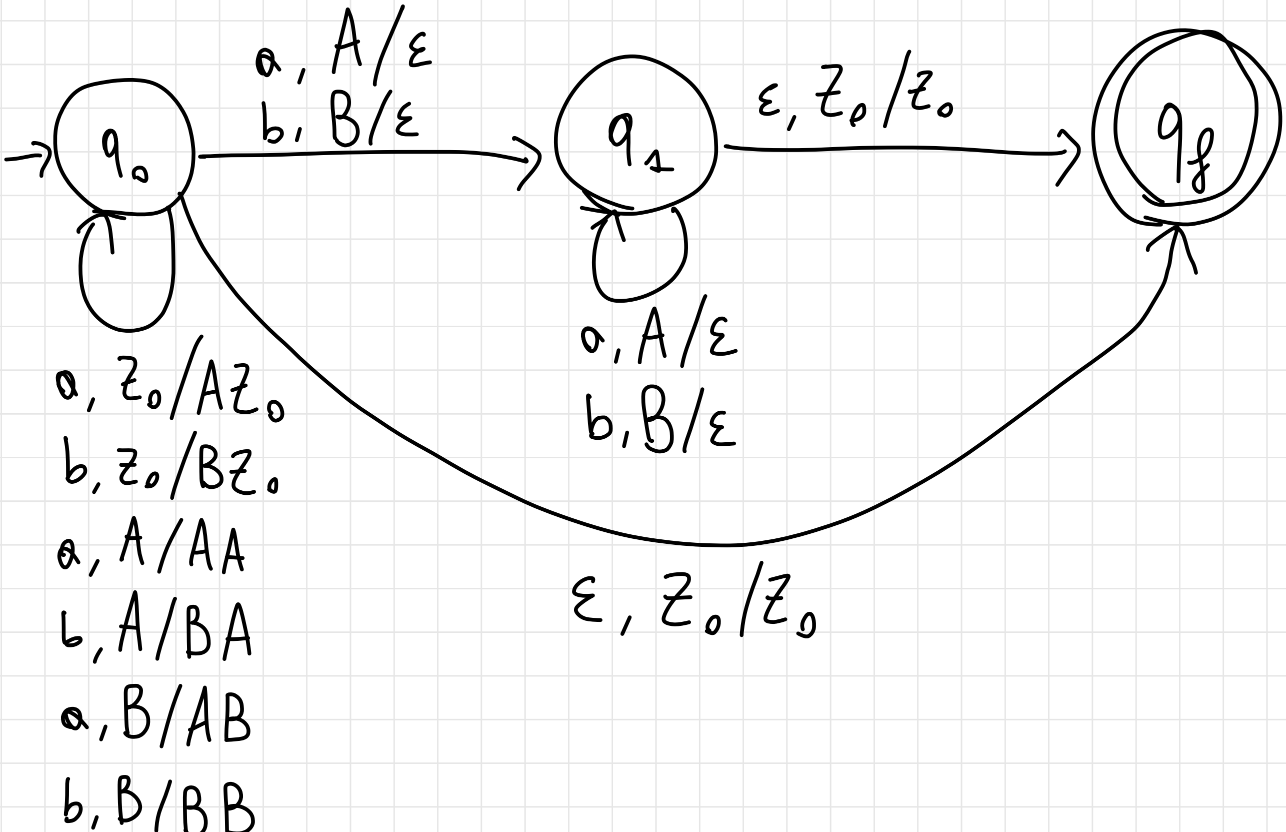
$$L = \underbrace{a^n b^n}_{PDA_1} \cup \underbrace{a^n b^{2n}}_{PDA_2}$$

non è riconoscibile da PDA.



es.: guessing

$L = \{ww^R \mid w \in \{a,b\}^*\}$



es.:

$$L = \{ wcw \mid w \in \{a,b\}^* \}^c$$

	c
FSA	✓
PDA	✓
NPDA	✗
TM	✗

$\underbrace{!\exists \;,\; TM \;,\; NPDA}$, ~~PDA~~ , ~~FSA~~

$$L = \{wcw\}^c \rightarrow PDA$$

$$\{wcw\} \rightarrow PDA \quad \text{assurdo}$$

$$L = \{ w c w \mid w \in \{a, b\}^* \}$$

$$V_1 \wedge V_2 \qquad \neg V_1 \vee \neg V_2$$

$$L_1 \cup L_2$$

$$L_1 = \left((a|b)^* \, c \, (a|b)^* \right)^c$$

abb.c.aab

$$L_2 = \underset{(a|b)^*}{\alpha} \, . \, c \, . \, \underset{(a|b)^*}{\beta} \qquad |\alpha| \neq |\beta|$$

abbcabb

w c w

$$L_3 = \alpha . a . \beta . c . \gamma . b . \delta \quad \longrightarrow \quad \text{NPDA}$$

$$\vee$$

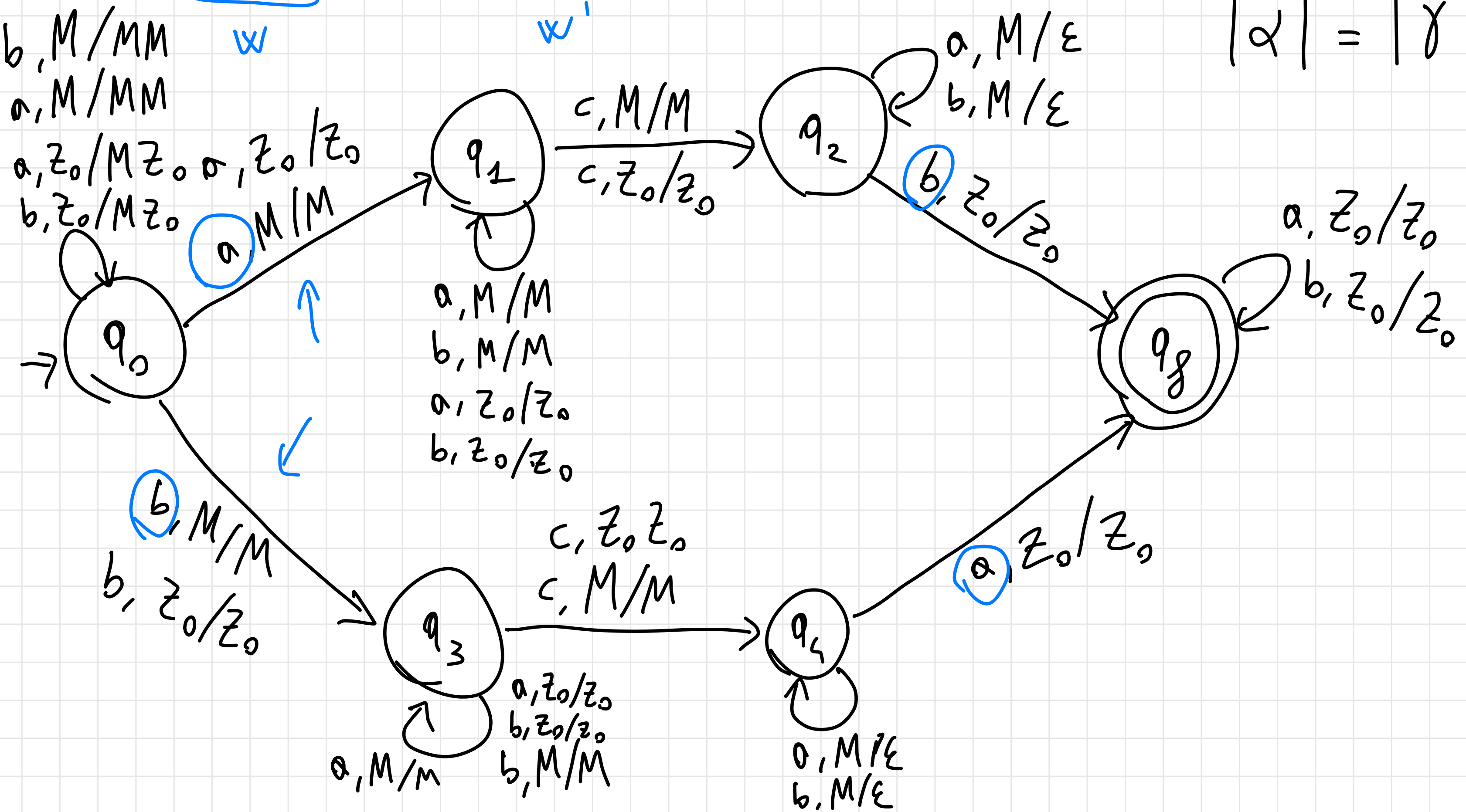
$$\alpha . b . \beta . c . \gamma . a . \delta \quad , \quad |\alpha| = |\gamma|$$

$$L_1 = \left((a|b)^* \, c \, (a|b)^* \right)^C \quad \longrightarrow \quad \text{FSA}$$

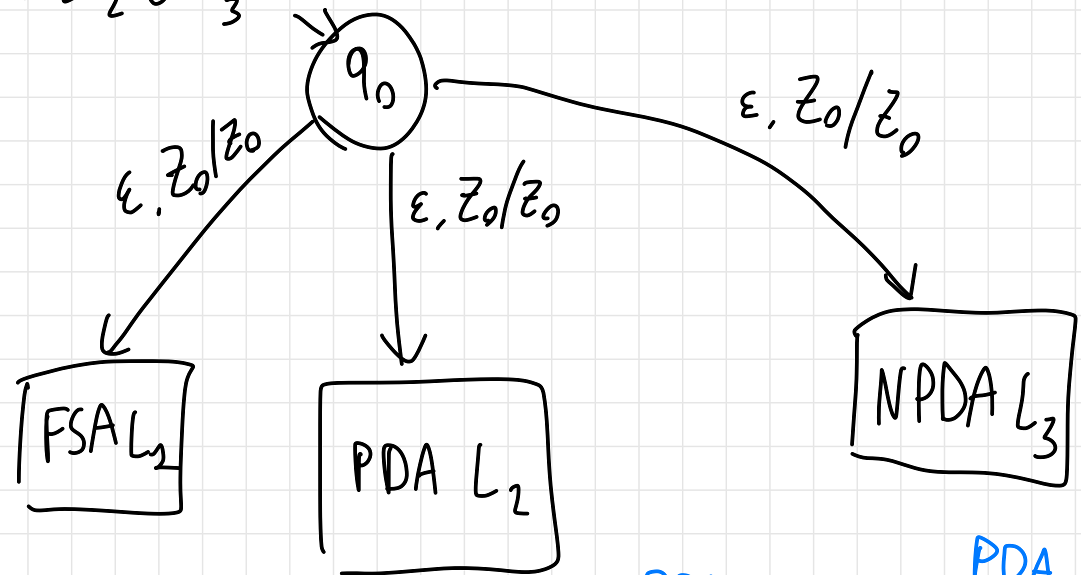
$$L_2 = \alpha . c . \beta \qquad \underbrace{|\alpha| \neq |\beta|} \quad \longrightarrow \quad \text{PDA}$$

$$L_3 = \underbrace{\alpha . a . \beta}_{w} . c . \underbrace{\gamma . b . \delta}_{w'} \;\vee\; \alpha . b . \beta . c . \gamma . a . \delta$$

$$|\alpha| = |\gamma|$$



$L : L_1 \cup L_2 \cup L_3$



NPDA

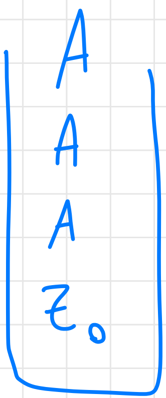
$$\overbrace{a^+ b^n c^n}^{\text{PDA}} \cap \overbrace{a^n b^n c^+}^{\text{PDA}}$$

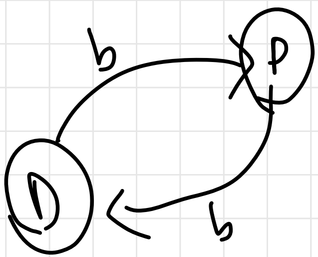
$$= a^n b^n c^n$$

$L_1 = \{a^n b^p b^p \mid n, p \geq 1\} \cup \{a^n b^p a^n \mid n, p \geq 1\}$

$a^n \; b^{2p}$

PDA





Idea: leggo le 'a', impilo 'A'

leggo le 'b', controllo (con gli stati) che siano pari

Se finisce la stringa: accetto se le 'b' pari

Se leggo altre 'a': spilo e controllo 'Z_0'

$$L_2 = \underbrace{\{a^n b^n \mid n \geq 1\}}_{\text{PDA}} \cup \underbrace{\{b^n c^n \mid n \geq 1\}}_{\text{PDA}} \quad \text{PDA}$$

$$L_3 = \underbrace{\{a^n b^n c^+ \mid n \geq 1\}} \cup \underbrace{\{a^+ b^n c^n \mid n \geq 1\}} \quad \text{NPDA}$$

L_1 : PDA

L_2 : PDA

L_3 : NPDA

$L_4 = L_1^c$ PDA

$L_5 = L_2^c$ PDA

$$L_3 = \{a^n b^n c^+ \mid n \geq 1\} \cup \{a^+ b^n c^n \mid n \geq 1\}$$ NPDA

$$L_6 = L_3^c = \{a^n b^n c^+ \mid n \geq 1\}^c \cap \{a^+ b^n c^n \mid n \geq 1\}^c$$

!∃

TM

NPDA

TM

$\#_a(w) \neq \#_b(w) \wedge \#_b(w) \neq \#_c(w)$

Grammatiche

$$G = \langle V_n , V_t , P , S \rangle$$

V_n: simboli non terminali

V_t: simboli terminali

P: produzioni

$S \in V_n$ assioma

$P \subseteq V_n^+ \times V^*$,

$V = V_n \cup V_t$

prod.

$$\underset{V_n^+}{AB} \longrightarrow \underset{V^*}{aBbB} \, ,$$

$V_n = \{A, B\}$

$V_t = \{a, b\}$

$\alpha \Rightarrow \beta \quad , \quad \alpha \in V^+ \quad , \quad \beta \in V^*$

$\alpha = \alpha_1 \underbrace{\alpha_2} \alpha_3 \quad , \quad \beta = \alpha_1 \underbrace{\beta_2} \alpha_3$

$\alpha_2 \longrightarrow \beta_2$

$A \longrightarrow a$

$\alpha = aAa$

$\beta = aaa$

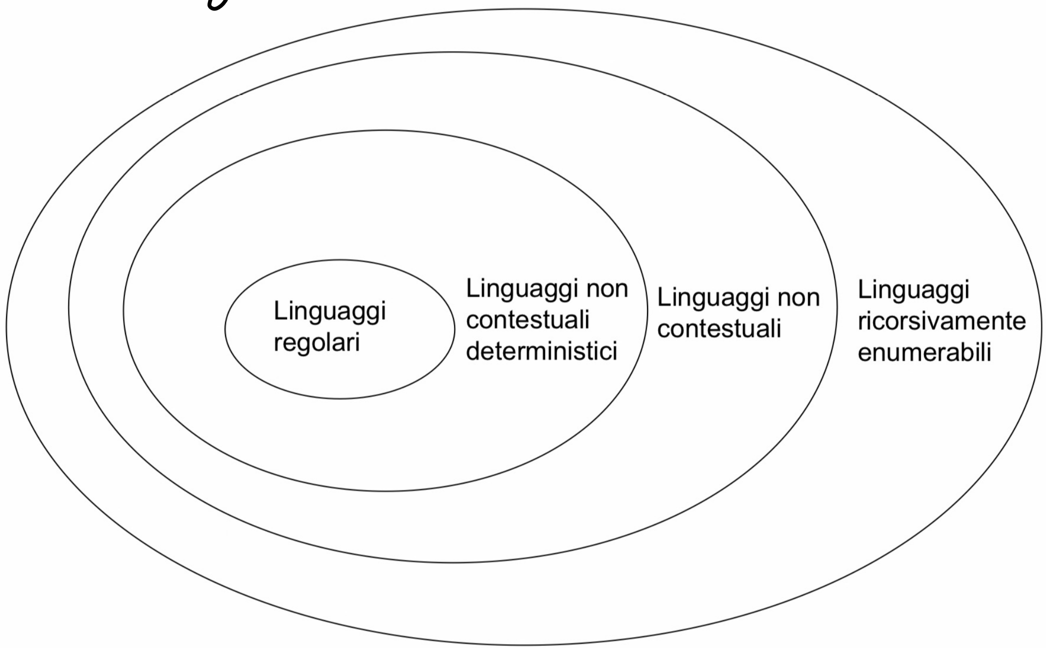
$\alpha \Rightarrow \beta$

$aAa \Rightarrow aaa$

Chiusura rifl. e tr. : $\stackrel{*}{\Rightarrow}$

$$L_G = \{ x \in V_t^* \mid \exists S \Rightarrow^* x \}$$

Gerarchia di Chomsky:



tipo			
3	Grammatica regolare : → FSA	$A \to aB \mid a \mid \varepsilon$	, $A \in V_n$ $a \in V_t$
2	Grammatiche Context-free : (CF) (libere dal contesto) → NPDA	$A \to \beta$	, $A \in V_n$ $\beta \in V^*$
0	Grammatiche generali :	$\alpha \to \beta$	, $\alpha \in V_n^+$ $\beta \in V^*$

$S \rightarrow aA$

è regolare? NO

$A \rightarrow Sb \mid b$

$S \Rightarrow aA \Rightarrow aSb \Rightarrow aaAb \Rightarrow aaSbb \Rightarrow$

$aaaAbb \Rightarrow aaabbb$

es.: $L = (aa)^*$

$\cancel{S \rightarrow aaS \mid \varepsilon}$

$S \rightarrow aA \mid \varepsilon$

$A \rightarrow aS$

$S \Rightarrow aA \Rightarrow aaS \Rightarrow aa$

$S \Rightarrow aA \Rightarrow aaS \Rightarrow aaaA$

$\Rightarrow aaaaS \Rightarrow aaaa$

es.:

$$L = \underbrace{a^n b^n}\ \underbrace{c^m\ a^m}, \quad n \geq 0, \quad m \geq 1$$

CF $\longleftrightarrow$ NPDA

$$S \Rightarrow S_1 S_2 \Rightarrow a S_1 b S_2 \Rightarrow$$

$$aa S_1 bb S_2 \Rightarrow aabb S_2 \Rightarrow$$

$$aabb\,ca$$

$$S \rightarrow S_1 S_2$$

$$S_1 \rightarrow a S_1 b \mid \varepsilon$$

$$S_2 \rightarrow c S_2 a \mid ca$$

es.: $L = L_\ell \cup L_b$ $\{0, 1, a\}$

$$L_\ell = \{ (n_\ell a^n)^+ \mid 0 \leq n_{(10)} \leq 3 \}$$

n little endian $10 = 1$ little endian

$$L_b = \{ (n_b a^n)^+ \mid 0 \leq n_{(10)} \leq 3 \}$$

n big endian $10 = 2$ big endian

$1 0 0 a a \in L_b$

$1 0 a \in L_\ell$

1

$L_b \cup L_\ell$

$(n_\ell a^n)^+ \cup (n_b a^n)^+ \quad , \quad n \in \{0,1\}^2$

$S \longrightarrow S_\ell \mid S_b$

$S_\ell \to 0Z \mid 1U$

$Z \to 0N_0 \mid 1N_2$

$U \to 0N_1 \mid 1N_3$

$N_3 \to aN_2 \qquad N_1 \to aN_0$

$N_2 \to aN_1 \qquad N_0 \to S_\ell \mid \varepsilon$

$S_b \to 0Z \mid 1U$

$Z \to 0N_0 \mid 1N_1$

$U \to 0N_2 \mid 1N_3$

$N_3 \to aN_2 \qquad N_2 \to aN_1$

$N_1 \to aN_0 \qquad N_0 \to S_b \mid \varepsilon$

$$S \to S_\ell \mid S_b$$

$$S_\ell \to 0 Z_\ell \mid 1 U_\ell$$

$$Z_\ell \to 0 N_{0\ell} \mid 1 N_{2\ell}$$

$$U_\ell \to 0 N_{1\ell} \mid 1 N_{3\ell}$$

$$N_{3\ell} \to a N_{2\ell} \qquad N_{1\ell} \to a N_{0\ell}$$

$$N_{2\ell} \to a N_{1\ell} \qquad N_{0\ell} \to S_\ell \mid \varepsilon$$

$$S_b \to 0 Z_b \mid 1 U_b$$

$$Z_b \to 0 N_{0b} \mid 1 N_{1b}$$

$$U_b \to 0 N_{2b} \mid 1 N_{3b}$$

$$N_{3b} \to a N_{2b} \qquad N_{2b} \to a N_{1b}$$

$$N_{1b} \to a N_{0b} \qquad N_{0b} \to S_b \mid \varepsilon$$

$$S \Rightarrow S_\ell \Rightarrow 0Z \Rightarrow 01N_2 \Rightarrow 01aaS_\ell \Rightarrow 01aa0Z \Rightarrow 01aa01N_1$$

$$\Rightarrow 01aa01a$$

es.: $L = x \in (a|b)^+ : \#_a(x) = \#_b(x)$

$S \to aSb \mid bSa \mid ab \mid ba$

baab

aabb

$S \Rightarrow aSb \Rightarrow abSab \Rightarrow abbaabba$

$S \Rightarrow bSa$

$S \to aGbG \mid bGaG$

abbaabba

$G \to aGbG \mid bGaG \mid \varepsilon$

aGbG

$S \rightarrow aGbG \mid bGaG$

$G \rightarrow aGbG \mid bGaG \mid \varepsilon$

abbaabba

$aGbG$

$S \Rightarrow aGbG$

a abab Gb   ababaabb

aGb

abab

$aGbG$

abbaabbaabab

$S \Rightarrow bGaG$

$\Rightarrow baG \Rightarrow baab$

Grammatica proposta a fine lezione:

$$L = x \in (a|b)^+, \quad \#_a(x) = \#_b(x)$$

$$S \to aGb \mid bGa \mid abG \mid baG$$

$$G \to aGb \mid bGa \mid abG \mid baG \mid \varepsilon$$

non può generare "aabbba"

non riesce a generare le due "a" iniziali vicine senza inserire due "b" in fondo