API - esercitazione 6

07/04/2021

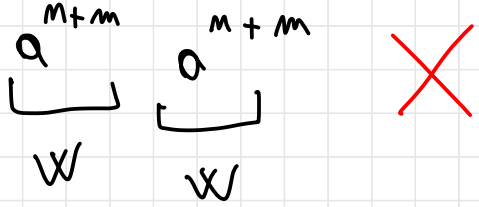
es. da ToE: si scrivano Automa e Gramm.

$$L = \{ a^n a^m a^m a^n \mid n, m \geq 1,\ n \text{ pari},\ m \text{ dispari} \}$$

NB.: FSA POA NPDA TM

Automa a pot. ric. min.

Gram. minor n° di prod.



$$L = \{ a^n a^m a^m a^n \mid n, m \geq 1,\ n \text{ pari},\ m \text{ dispari} \}$$

$$a^{n+m+m+n} = a^{2(n+m)}$$

n pari
m dispari

$n = 2 \qquad m = 1$

$$a^{2(3)} = a^6$$

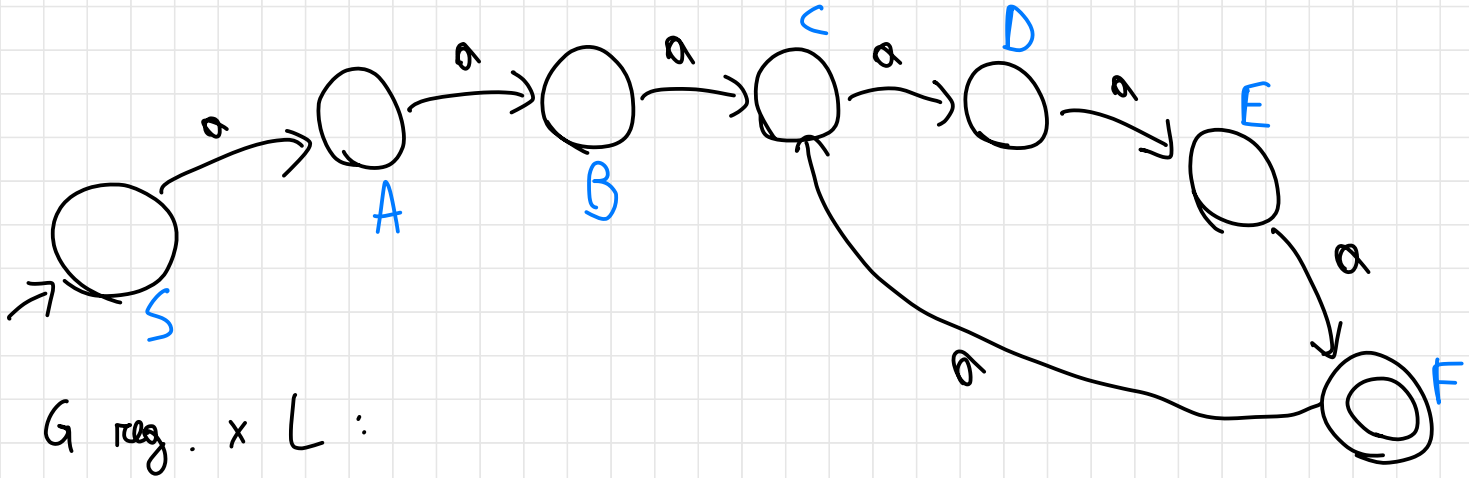
$n = 2 \qquad m = 3$

$$a^{2(2+3)} = a^{10}$$

$$a^{14}, a^{18}, \ldots$$

$$a^{2(m+m)} = a^{2(2k+1)}, \quad k \geq 1$$

$$a^{2(2k+1)} = a^{4k+2} = a^{2+4k}$$



G reg. x L:

$S \to aA \quad A \to aB \quad B \to aC \quad C \to aD \quad D \to aE$

$E \to aF \quad F \to aC \mid \varepsilon$

$S \to a^6 A$

$A \to a^4 A \mid \varepsilon$

$S \to aaaaS \mid a^6$

$S \Rightarrow a^4 S \Rightarrow a^4 a^4 S \Rightarrow a^4 a^4 a^6$

$S \Rightarrow a^6$

es TdE:

G:

$S \to AB$

$A \to \varepsilon \mid CAC \mid D$

$D \to aDa \mid \varepsilon$

$C \to cC \mid Cc \mid \varepsilon$

$B \to bbB \mid b$

Trovare Automa (a pot. min.)

10 (7 se l'automa non è a pot. min.)

D: che tipo è G? CF

$S \to AB$

$A \to \varepsilon \mid CAC \mid D$

$D \to aDa \mid \varepsilon$

$C \to cC \mid Cc \mid \varepsilon$

$B \to bbB \mid b$

$S \Rightarrow AB \Rightarrow CACB$

$\overset{(n-1)}{\Rightarrow} C^n A C^n B$

$\Rightarrow C^n D C^n B$

$\overset{m}{\Rightarrow} C^n a^m D a^m C^n B$

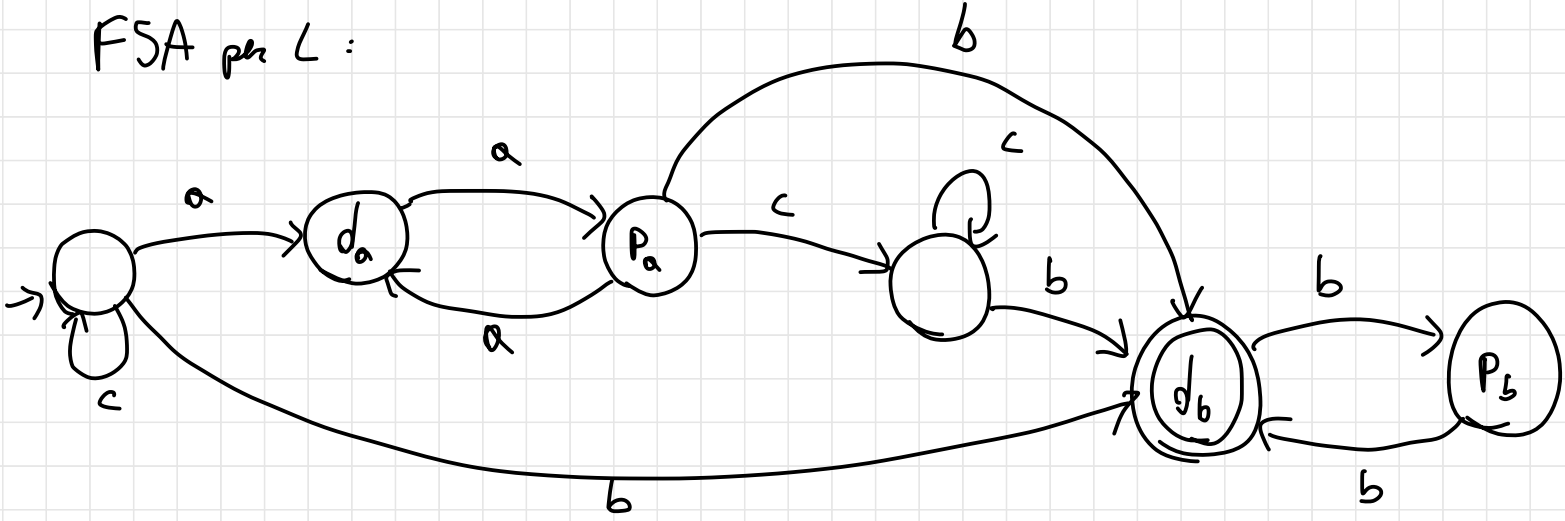
$\Rightarrow C^n a^m a^m C^n B \overset{*}{\Rightarrow} c^* a^m a^m c^* B \overset{*}{\Rightarrow} c^* a^m a^m c^* b^{2k+1}$

$CCC\, a^m a^m\, CCCB \Rightarrow cC\, CC\, a^m\, a^m\, CCCB$

$\Rightarrow cc\, C\, CC\, a^m\, a^m\, CCCB \overset{*}{\Rightarrow} c^{1000}\, CCC\, a^m\, a^m CCCB$

$$L = c^* \underbrace{a^m a^m}_{a^{2m}} c^* b^{2k+1} \qquad m \geq 0 \;,\; k \geq 0$$

FSA per L:



NFSA per L andava bene comunque

es TdE: $\forall x \left(x \in L \Leftrightarrow \exists y \left(x = y\, b^n\, y \wedge y \in L' \wedge n > 0 \right) \right)$

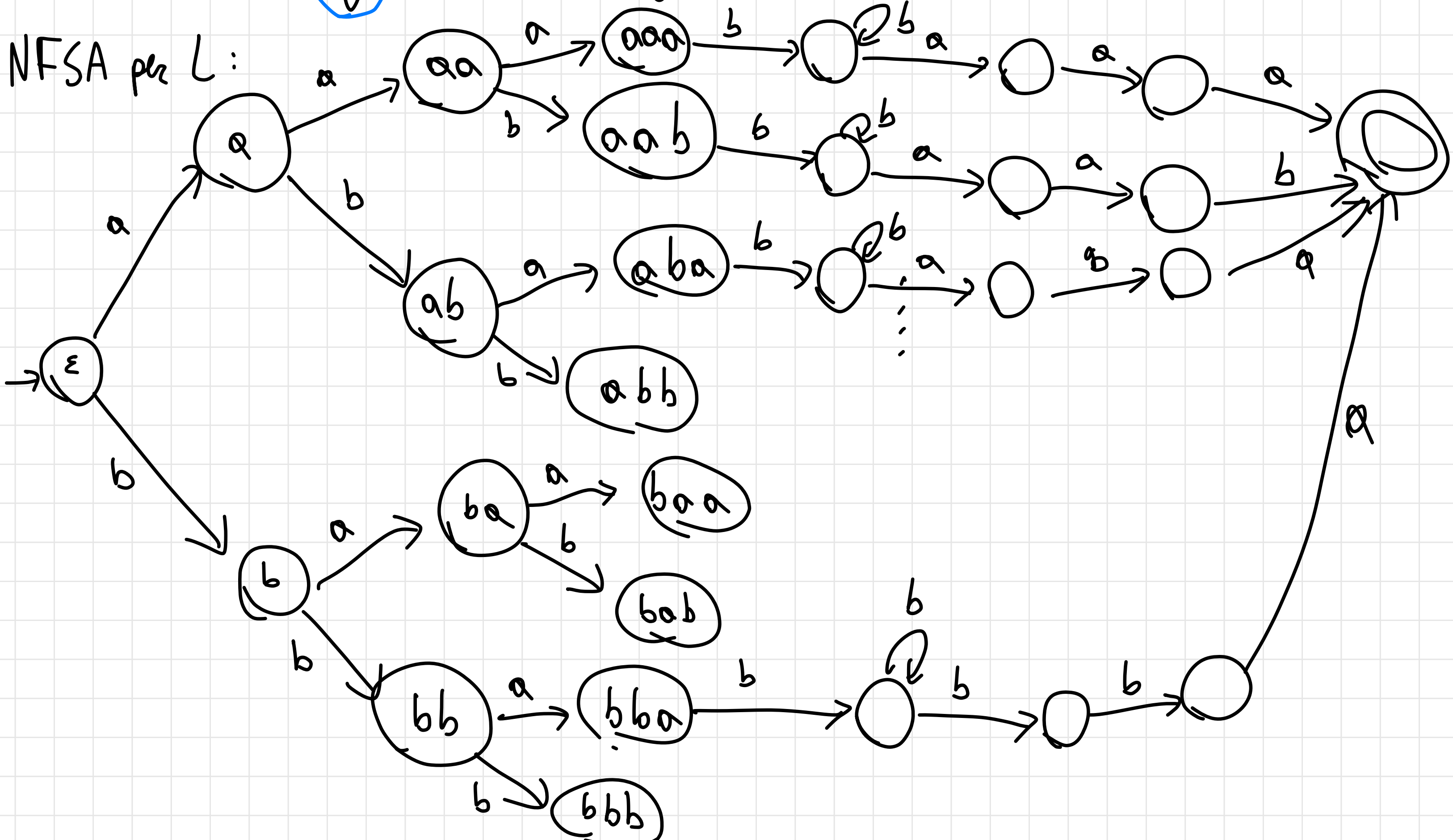
Descrivere l'automa a pot. min. per riconoscere L nei seguenti casi:

(1) $L' = \{ x \in \{a,b\}^* \mid |x| = 3 \}$

$x = y\, b^n\, y$, $y \in \{a,b\}^* \wedge |y| = 3$, $n > 0$

$x = y\, b^n\, y \quad , \quad y \in \{a,b\}^* \wedge |y| = 3 \quad , \quad n > 0$

NFSA per L:

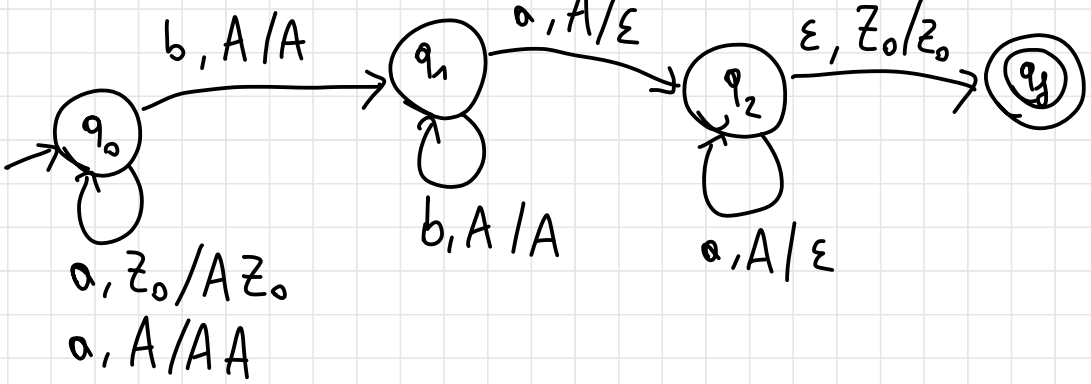


$$\forall x \left(x \in L \Leftrightarrow \exists y \left(x = y\, b^m\, y \wedge y \in L' \wedge m > 0 \right) \right)$$

② $L' = \{ a^m \mid m > 0 \}$

$x = a^m \; b^n \; a^m$, $n, m > 0$

PDA per L:



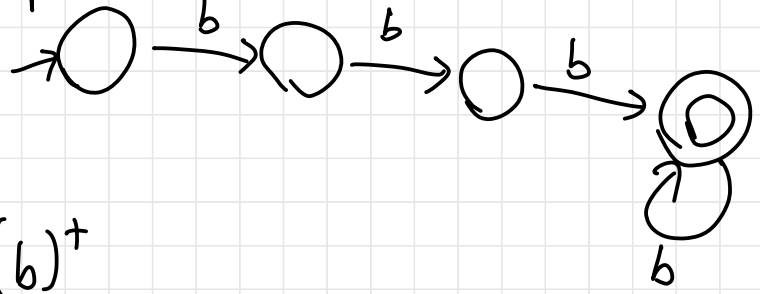
$$\forall x \left(x \in L \Leftrightarrow \exists y \left(x = y\, b^m\, y \wedge y \in L' \wedge m > 0 \right) \right)$$

③ $L' = \{ b^m \mid m > 0 \}$

$$x = y\, b^m\, y = b^m\, b^n\, b^m \quad , \; n, m > 0$$

$$= b^{2m+n}$$

FSA für L:



$bbb\,(b)^*$ $\qquad$ $bb\,(b)^+$

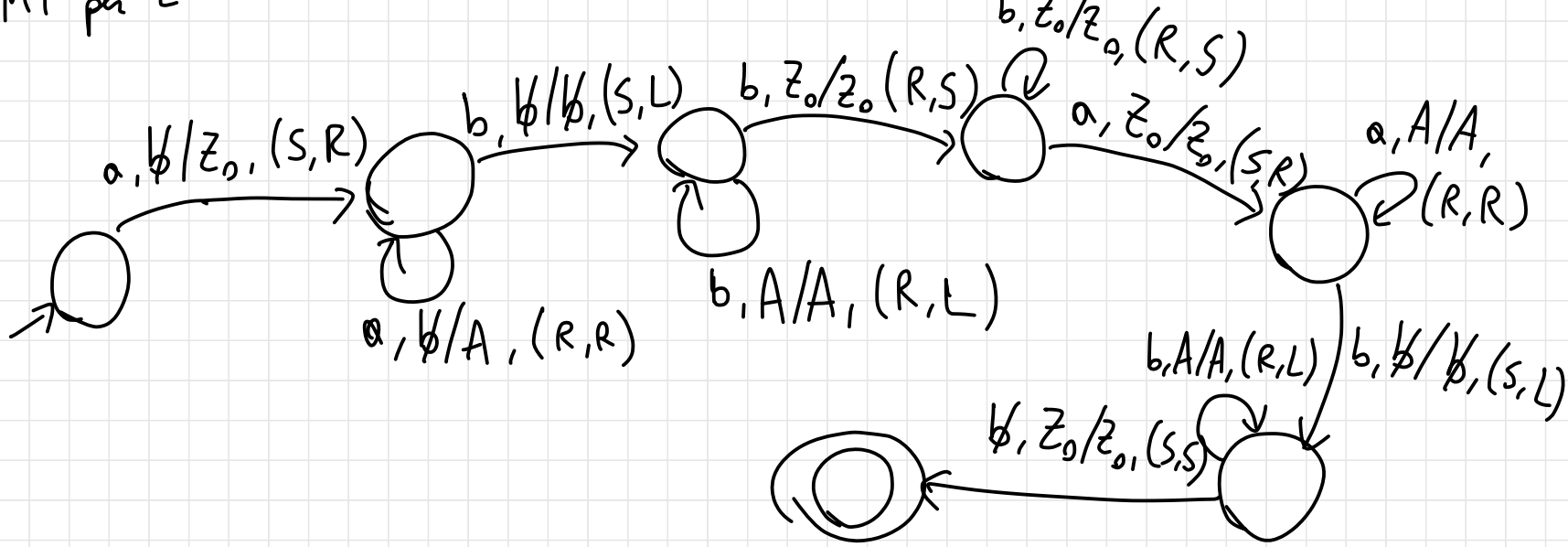
$(bbb)^+$

$$\forall x \left(x \in L \Leftrightarrow \exists y \left(x = y\, b^m\, y \wedge y \in L' \wedge m > 0 \right) \right)$$

④ $L' = \{ a^m b^m \mid m > 0 \}$

$x = a^m b^m \; b^n \; a^n b^n, \quad n, m > 0$

MT per L:



es. ToIE: Data una Grammatica trovare automa (FSA, PDA, NPDA, TM) a pot. min. che riconosca $L(G)$

(1) $S \to aXb \mid bXa$

$X \to aXb \mid bXa \mid c$

$baaXbba$

$\downarrow$

$w c (w^R)^{-1}$

non standard

$b^{-1} = a$

$a^{-1} = b$

$S \Rightarrow bXa \Rightarrow baXba \Rightarrow baaXbba \Rightarrow baaaXbbba$

$\Rightarrow baaa c bbba$

$S \Rightarrow aXb \Rightarrow abXab \Rightarrow abaXbab \Rightarrow aba c bab$

a b c a b

a b c b a a b c a b

a a b c a b b → PDA

B
A
A
Z_0

(2)

$S \to X \mid Y$

$X \to aYb \mid ab$

$Y \to bXa \mid ba$

$S \Rightarrow X \Rightarrow aYb \Rightarrow abXab \Rightarrow abaYbab \Rightarrow ababab$

$S \Rightarrow Y \Rightarrow bXa \Rightarrow baYba \Rightarrow babXaba \Rightarrow babaYbaba$

$\Rightarrow bababXababa \Rightarrow bababababababa$

$(ab)^+ \mid (ba)^+$

ababab

bababa

$S \rightarrow X \mid Y$

$X \rightarrow aYb \mid ab$

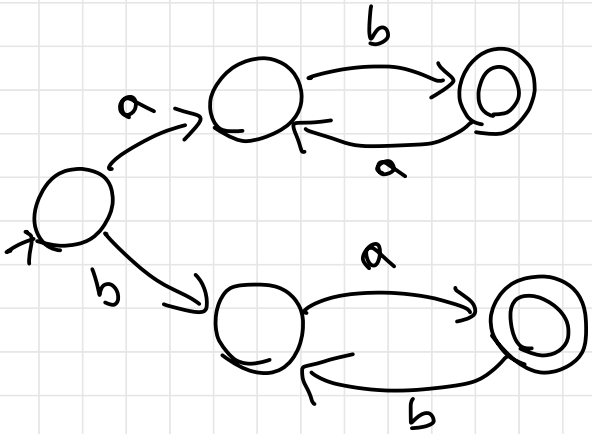
$Y \rightarrow bXa \mid ba$

$S \Rightarrow X \Rightarrow aYb$

$\Rightarrow abab$

G è CF

$L = (ab)^+ \mid (ba)^+$ → FSA



(3) $S \to aXb \mid bYa$ (G è CF)

$X \to aSb \mid ab$

$Y \to bSa \mid ba$

→ NPDA

$S \Rightarrow aXb \Rightarrow aabb$

$\Rightarrow aaSbb \Rightarrow aaaXbbb \Rightarrow aaaaSbbbb$

$\Rightarrow aaaabYabbbb \Rightarrow aaaabbSaabbbb$

$\Rightarrow aaaabbaXbaabbbb \Rightarrow aaaabbaabbaabbbb$

$w(w^R)^{-1}$, $w \notin \{a,b\}^*$ $w \in \{aa, bb\}^+$

④

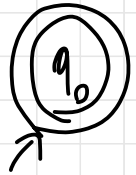
$S \to X \mid Y$

$X \to aYb \mid bYa$

$Y \to bXa \mid aXb$

$S \Rightarrow X \to aYb \Rightarrow abXab \Rightarrow^{*}$

$L = \emptyset$ → FSA



✗

accetta ε

$L' = \{\varepsilon\}$

es. TdE: $L = \{ a^n b^m , n > 0 , m = f(n) \}$

(A) trova $f(n)$ per cui L sia generabile da gramm. reg.

$$L = \{ a^n b^{f(n)} , n > 0 \}$$

$f(n) = k$ $\quad L = \{ a^n b^k , n \geq 0 \}$

$f(n) = n \rightarrow a^n b^n$ CF!

$S \rightarrow aSb \mid ab$

B trova $f(n)$ per cui L sia generabile da gramm. CF

$$L = \{ a^n b^{f(n)} \quad , \; n > 0 \}$$

$f(n) = k\,n$

$a^n \; b^{2n+3}$

(c)

$$f(n) = n^2$$

$$a^n \, b^{n^2} = a^n \, b^{n \cdot n}$$

(vedi esercitazione TM)

es. T o I E : Alfabeto $\{a, b\}$

$L = bbwbb$, $w \in (ba)^+$

(1) Grammatica che gen. L con minor n° possibile di non term.

bb ba bb

bb ba ba ba bb

⋮

$S \to bbA$

$A \to baA \mid bb$

X

$S \Rightarrow bbA$

$\Rightarrow bbbb$

$S \to bbCbb$

$C \to baC \mid ba$

✓

CF

$S \to bb\,A\,bb$

$A \to bAa \mid \varepsilon ba$ ✗

$(ba)^+$

$b^n a^n$

$S \Rightarrow bb\,A\,bb \Rightarrow bb\,b\,A\,a\,bb \Rightarrow bb\,bb\,A\,aa\,bb$

$\Rightarrow bb\underbrace{bb\,ba\,aa}\,bb$

$b^n a^n$

$\neq$

$(ba)^+$

(2) La gramm. def. al punto (1) è a pot. min. ? No

$bb(ba)^+bb$

Trova gramm. a pot. min. x L.

$S \to bA$

$A \to bB$

$B \to bC$

$C \to aD \mid aB$ ($(ba)^+$)

$D \to bE$

$E \to b$

(3) Automa per $L' = bb\,w\,bb\,w\,bb$, $w \in (ba)^+$

(pot. ric. min.)

M
M
M
M
Z_0

NPDA o PDA?

Idea: leggo 'bb';

leggo 'ba', impilo 'M'

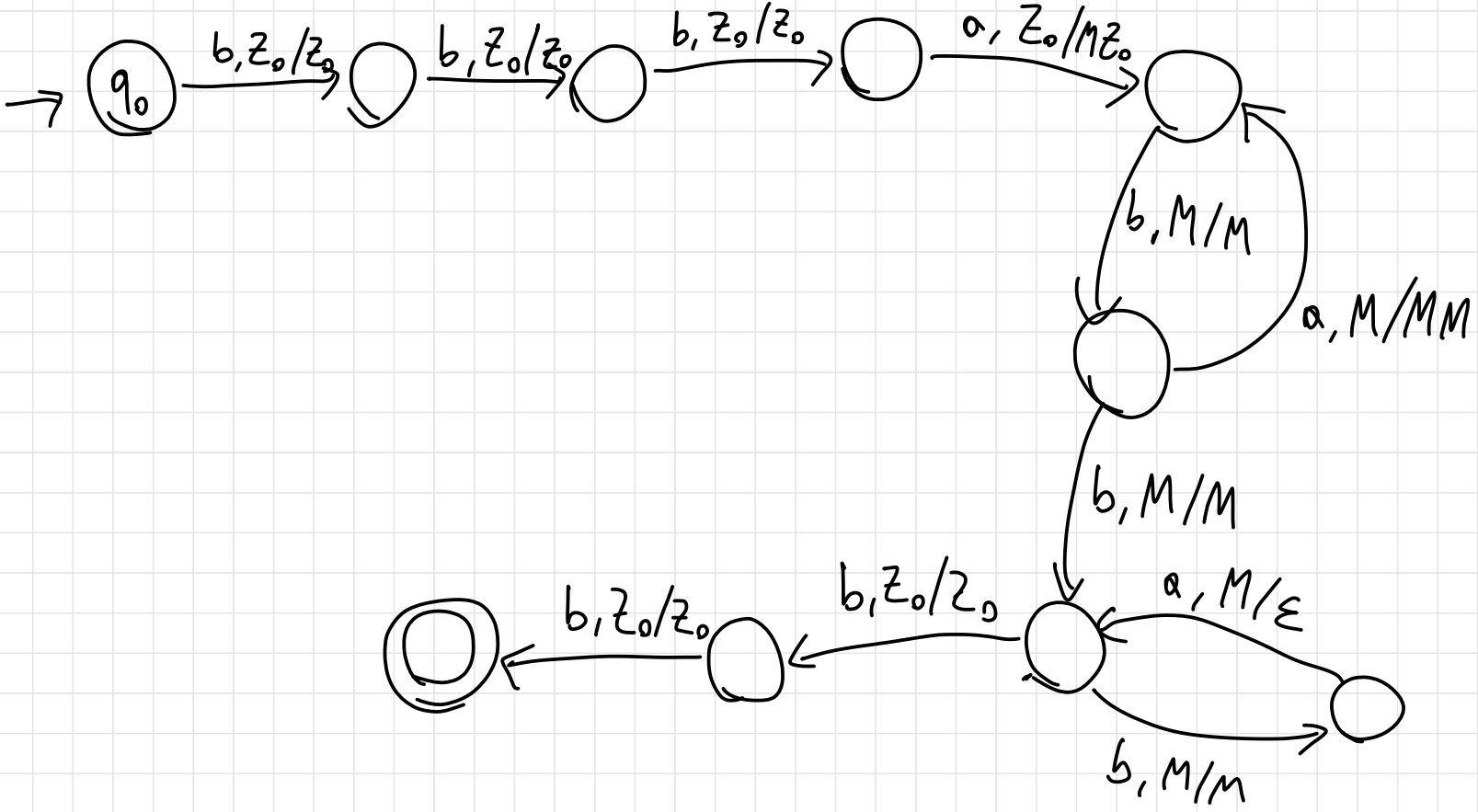
leggo 'b': se segue 'a' → impilo 'M'

se segue 'b' → aspetto di leggere w

leggo 'ba', spilo 'M'

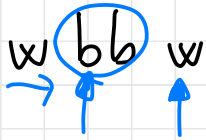
Z_0 in pila → 'bb'

PDA (det.) × L :



Automa per $L' = bb\,w\,bb\,w\,bb$, $w \in (ba)^+$

(pot. ric. min.)



$S \rightarrow bb\,ba\,A\,ba\,bb$

$A \rightarrow ba\,A\,ba \mid bb$

$S \Rightarrow bb\,ba\,A\,ba\,bb$

$\Rightarrow^* bbbabababbbabb$

Es TdE: Gramm. per L:

$$L = \{ x \in \{a,b,c\}^* \mid \#_a(x) = 2n,\ \#_b(x) = 4,\ \#_c(x) = n,\ n > 0 \}$$

$$S \to AACX$$

$$X \to AACX \mid BBBB \qquad S \Rightarrow^* (AAC)^+ BBBB$$

$$AB \to BA$$

$$BA \to AB$$

$$AC \to CA \qquad BC \to CB$$

$$CA \to AC \qquad CB \to BC$$

swap rule

$$A \to a$$

$$B \to b$$

$$C \to c$$